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A $J^\pi = 0^-$ resonance for $\pi NN \to \pi NN$ in the Skyrme model

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ABSTRACT

The scattering of pions on a dibaryon configuration is analyzed within the $SU(2)$ Skyrme model. It is shown that this model leads to a low-lying $(J^P, I) = (0^-, 2)$ resonance. The possibility that this resonance corresponds to one proposed recently in the context of double charge exchange pion scattering on nuclei is discussed. Given the setup used in those experiments we also show that a resonance with isospin assignment $I = 0$ cannot be excited according to the description presented here.

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A recent reevaluation of (π^+, π^-) double charge exchange cross sections over a range of nuclei has led to experimental evidence for a $J^P = 0^-$ resonance of the two nucleon system only 50 MeV above pion threshold, i.e. far below the delta resonance[1]. Its width has been estimated to amount to a few MeV only. At present, it appears that this reaction produces the only clear candidate for a dibaryon resonance standing out from a multitude of unclear signatures in various channels. Although the arguments given in Refs.[1, 2] seems to favor the assignment $I = 0$ for the isospin of the resonance, the case $I = 2$ is not completely excluded[3].

Theoretical considerations of this specific resonance have been performed in terms of the non-relativistic quark model assuming its structure could be attributed to p-wave excitations of six valence quarks[4]. Not surprisingly, the conclusion there is essentially negative, since its description is the analogue of odd parity excitations in non-relativistic three quark systems located around 600 MeV excitation energy in the latter.

The Skyrme model, finally, is known to exhibit a non-trivial winding number two configuration, the torus[5, 6], which has had, until recently, a rather unclear interpretation ranging from "an artifact of the model" to "the origin of intermediate nucleon-nucleon attraction", for the most ambitious effort in latter direction see refs.[7, 8]. A recent investigation of the baryon number two configurations in large N_C chiral perturbation theory[9] has, however, shown that the meson cloud around two explicit baryon sources indeed will assume the shape of a torus, thus adding some weight to such toroidal configurations.

It is the purpose of the present letter, to examine the scattering of pions from a torus-like meson cloud much in the spirit of pion-soliton scattering[10, 11], which has shown remarkable success for cases where $1/N_C$ corrections are small: in those cases the leading N_C contribution suffices and explains gross features of physical pion-nucleon scattering in the real world at $N_C = 3$. It is our conjecture, that the leading N_C terms in the winding number two sector, which firstly lead to a toroidal configuration at small internucleon distances[9], will also be able to capture the essence of this postulated resonance.

The chiral fields of the axially symmetric $B = 2$ configuration of lowest energy, the torus, follow from a specific ansatz [12, 5, 6]

$$U_T(\mathbf{x}) = \exp\{i\boldsymbol{\tau}\cdot\mathbf{n}\,\chi\} \tag{1}$$

where the chiral angle $\chi = \chi(r, \theta)$ satisfies the boundary conditions

$$\chi(r = 0, \theta) = \pi, \qquad \chi(r \to \infty, \theta) = 0, \tag{2}$$

and the orientation $\mathbf{n}(\mathbf{x})$ of the pion fields

$$\mathbf{n} \;=\; \begin{pmatrix} \cos 2\varphi \, \sin\alpha(r,\theta) \\ \sin 2\varphi \, \sin\alpha(r,\theta) \\ \cos\alpha(r,\theta) \end{pmatrix} \tag{3}$$

rotates azimuthally twice as fast as for hedgehog configurations. The symmetries of the torus are given by

$$\begin{aligned} \chi(r,\theta) &= \chi(r,\pi-\theta), \\ \alpha(r,\theta) &= \pi - \alpha(r,\pi-\theta). \end{aligned} \tag{4}$$

Chiral fields with respect to the physical isospin axes are obtained after an isospin rotation $A \in \mathrm{SU}(2)$

$$\boldsymbol{\tau}\cdot\mathbf{n}(\mathbf{x})\,\chi(\mathbf{x}) \to A\boldsymbol{\tau}A^\dagger \cdot \mathbf{n}(R^{-1}\cdot\mathbf{r})\,\chi(R^{-1}\cdot\mathbf{r}) \tag{5}$$

and the spatial coordinates $\mathbf{r}$ in the laboratory system are related to the body-fixed coordinates $\mathbf{x}$ by a rotation R. The construction of the rotational states of the torus[12] follows the collective quantization procedure $A = A(t), R = R(t)$ and leads to a tower of states

$$\langle A, R \mid T\,T_3, \Sigma\,S_3, N_3\rangle = (-)^{T-T_3} D^T_{-T_3\,N_3}(A) D^\Sigma_{2N_3\,S_3}(R) \tag{6}$$

where the symmetries $S_3^{bf} = 2T_3^{bf}$ of the torus require that body-fixed spin be twice as large as body-fixed isospin. Under a parity transformation the chiral fields in the exponent transform according to

$$\mathbf{P}(\mathbf{n}(\mathbf{x})\,\chi(\mathbf{x}))\mathbf{P}^{-1} = -\mathbf{n}(-\mathbf{x})\,\chi(-\mathbf{x}) = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \cdot \mathbf{n}(\mathbf{x})\,\chi(\mathbf{x}) \tag{7}$$

thus being equivalent to an isospin-rotation around the 3-axis by an angle of π. This leads to the conclusion that the parity of a given rotator state is carried by the projection of body-fixed isospin on the symmetry axis of the torus

$$P_{rot} = (-)^{N_3} \quad . \tag{8}$$

From the spins, isospins and parities of the rotational excitations we conclude, that there is no state of negative parity and spin zero present in purely rotational excitations of the torus. These additional degrees of freedom can only come from intrinsic vibrational excitations to which we turn now.

Small amplitude oscillations of the torus are conveniently parametrized by

$$U_{T^*}(\mathbf{x}) \quad = \quad \exp\{i\boldsymbol{\tau}\cdot(\mathbf{n}\,\chi \quad + \boldsymbol{\eta})\} \tag{9}$$

where the fluctuations can always be written in terms of a scaling ansatz [13]

$$\boldsymbol{\tau}\cdot\boldsymbol{\eta} = \sum_M (-)^M \tau_{-M}\Big(\mathbf{s}(\mathbf{x},t)\cdot\nabla\Big)^P_{K_3} n_M(\hat{x})\chi(\mathbf{x}) \tag{10}$$

for the fluctuating fields. A complete set of displacement fields $\mathbf{s}$ can be constructed from a set of three basis vectors

$$\{\mathbf{s}(\mathbf{x})\} = \{\mathbf{x}, \nabla, \mathbf{x}\times\nabla\}\sum_K Y^K_{K_3}(\hat{x})F_K(r)\,. \tag{11}$$

multiplied by a harmonic time dependence.
The additional parity P carried by the vibration is determined by the parity of the scaling operator:

$$\mathbf{P}\left(\mathbf{s}(\mathbf{x})\cdot\nabla\right)^P_{K_3}\mathbf{P}^{-1} = -\left(\mathbf{s}(-\mathbf{x})\cdot\nabla\right)^P_{K_3} = P\left(\mathbf{s}(\mathbf{x})\cdot\nabla\right)^P_{K_3}. \tag{12}$$

Since the torus has no spherical symmetry the displacement fields of different multipolarity K do not decouple, however, their projection K_3 on the symmetry-axis does. Arbitrary displacement fields can therefore be expanded into an infinite sum of multipoles L

$$\left(\mathbf{s}(\mathbf{x})\cdot\nabla\right)^P_{K_3} n_M(\hat{\mathbf{x}})\chi(\mathbf{x}) = \sum_L F^L_{K_3\,P\,M}(r)\,Y^L_{K_3+2M}(\hat{x})\,. \tag{13}$$

Generally, soliton fluctuations will be unbound, i.e. above pion threshold, in which case the displacements fields must grow exponentially with $m_\pi r$ in order to compensate for the exponential decrease of the chiral angle χ. However, for an estimate of the resonance energies, a polynomial form for the displacement fields that keeps the vibrations localized in the vicinity of the torus can be used[13]. In our numerical calculations below we will use such an approximation.

Since the $\mathbf{x}$ in e.g. eq.(13) are the components of coordinates with respect to the body-fixed symmetry axis of the torus they do not contribute to the spin of the configuration and since the isospin matrices in eqs.(9,10) also refer to the body-fixed axes they do not contribute to isospin. Total spin $J\,J_3$ and total isospin $I\,I_3$ of the configuration of chiral fields for the vibrating torus must be carried by the Euler angles[11] which indicate the orientation of the laboratory coordinates $\mathbf{r} = R\cdot\mathbf{x}$ relative to the body-fixed ones and of the isorotation A necessary to transform

body-fixed matrices to the laboratory $\boldsymbol{\tau} \to A\boldsymbol{\tau}A^\dagger$. For a rigidly rotating body the (unnormalized) quantized states are given by a generalization of eq.(6)

$$\langle A, R \mid I\, I_3, J\, J_3\rangle = (-)^{I-I_3} D^I_{-I_3\, N_3}(A)\, D^J_{2N_3+K_3\, J_3}(R) \tag{14}$$

where the indices N_3 and $2N_3 + K_3$ are fixed by the symmetry constraints of the rotating object as will be seen later. The full expression for the body-fixed fluctuations is summarized by

$$\boldsymbol{\tau}\cdot\boldsymbol{\psi} = \sum_M (-)^M \tau_{-M} \Big\{ \Big(\mathbf{s}(\mathbf{x},t)\cdot\nabla \Big)^P_{K_3} n_M(\hat{x})\chi(\mathbf{x}) \Big\} \Big\{ (-)^{I_3} D^I_{-I_3\, N_3}(A)\, D^J_{2N_3+K_3\, J_3}(R) \Big\}\,. \tag{15}$$

Expressed in terms of space coordinates $\mathbf{r}$ of the laboratory system the scaling vibration reads

$$\begin{aligned} (\mathbf{s}(\mathbf{x})\cdot\nabla)^P_{K_3}\, n_M(\hat{x})\chi(\mathbf{x}) &= \sum_L F^L_{K_3\, P\, M}(r)\, Y^L_{K_3+2M}(\hat{x}) \\ &= \sum_{L\, L_3} F^L_{K_3\, P\, M}(r)\, (-)^{K_3+2M-L_3} D^L_{-2M-K_3 \quad -L_3}(R)\, Y^L_{L_3}(\hat{r})\,. \end{aligned} \tag{16}$$

We now rotate the body-fixed fluctuation in eq.(15) to physical isospin axes, i.e. to the laboratory system, also inserting eq.(16)

$$\begin{aligned} \boldsymbol{\tau}\ \cdot\ \boldsymbol{\varphi} &= A\boldsymbol{\tau}\cdot\boldsymbol{\psi}A^\dagger \\ &= \sum_{m\, M} \tau_m D^1_{m\, -M}(A)(-)^M \Big\{ \sum_{L\, L_3} F^L_{K_3\, P\, M}(r)\, (-)^{K_3+2M-L_3} D^L_{-2M-K_3 \quad -L_3}(R)\, Y^L_{L_3}(\hat{r}) \Big\} \\ &\qquad \times \Big\{ (-)^{I_3} D^I_{-I_3\, N_3}(A)\, D^J_{2N_3+K_3\, J_3}(R) \Big\}\,. \end{aligned} \tag{17}$$

Generally the energies of the vibrations are above pion threshold and thus in the continuum: in this case we speak of pion-torus scattering at fixed total spin and isospin. In order to make the different scattering channels more explicit we can recouple the D-functions of same Euler angles in eq.(17) to a single D-function, which then will represent the two-baryon target state coupled with isospin one and angular momentum L of the scattered pion:

$$\begin{aligned} \boldsymbol{\tau}\cdot\boldsymbol{\varphi} \ =\ & (-)^{K_3} \sum_{L\,\Sigma\, T\, M} (-)^{I-T+L-M} \frac{\hat{\Sigma}\hat{T}}{\hat{J}\hat{I}} \\ & \left(\begin{smallmatrix} 1 & I \\ M & N_3 \end{smallmatrix}\Big|\begin{smallmatrix} T \\ N_3+M \end{smallmatrix}\right) \left(\begin{smallmatrix} L & J \\ -K_3+2M & 2N_3+K_3 \end{smallmatrix}\Big|\begin{smallmatrix} \Sigma \\ 2N_3+2M \end{smallmatrix}\right) F^L_{K_3\, P\, -M}(r) \\ & \Big[\boldsymbol{\tau}\circ D^T_{\cdot\, N_3+M}(A)\Big]_{I\, I_3} \Big[D^\Sigma_{2N_3+2M\, \cdot}(R)\circ Y^L_{\cdot}(\hat{r})\Big]_{J\, J_3}\,. \end{aligned} \tag{18}$$

In eq.(18) we can identify the different scattering channels

$$\Big[\boldsymbol{\tau}\circ D^T_{\cdot\, N_3+M}(A)\Big]_{I\, I_3} \Big[D^\Sigma_{2N_3+2M\, \cdot}(R)\circ Y^L_{\cdot}(\hat{r})\Big]_{J\, J_3} \tag{19}$$

where pionic isospin one and target isospin T are coupled to total $I\,I_3$ and pionic angular momentum L with target spin Σ to total $J\,J_3$. All target states in these coupled channels obey the symmetry constraints of the torus: the third component of body-fixed spin must be twice as large as body-fixed isospin. The constraint is satisfied because we had made the correct choice for the indices of the Euler angle wavefunctions at the beginning in eq.(14). When the torus is quantized according to two identical fermions an additional constraint on the target

$$(-)^{\Sigma+T} = -1 \tag{20}$$

emerges because of the Pauli exclusion principle. From the full expression for the fluctuations around the torus we see that there is no restriction on the multipolarity L of the pion and the target spins Σ,

$$\mid L - \Sigma \mid \leq J \leq L + \Sigma \quad , \tag{21}$$

quite in contrast to the $B = 1$ Skyrmion fluctuations. There the possible isospins of the target $\mid T - 1 \mid \leq I \leq T + 1$ limit the possible spins because hedgehogs obey the stronger symmetry constraint $T = \Sigma$.

Since the right index N_3 of the rotator state in eq.(14) carries parity $(-)^{N_3}$ we can read off

$$P_{\text{total}} = (-)^{N_3}\, P \tag{22}$$

as total parity which will label the scattering process considered.

Having given the general expressions corresponding to the scattering of pions on the torus we will now study the particular configurations used in the double charge exchange experiments. The experimental setup has singled out one specific incoming channel where an $L = 0$ pion is incident an a two-nucleon pair in a spin $\Sigma = 0$ state. This spin was chosen by the fact that the two nucleons, being close together, cannot have any relative angular momentum and their isospin must be $T = 1$ in double charge exchange experiments. Since total spin $J = 0$ was observed by the angular distributions of the cross sections the incoming pionic angular momentum is fixed.

Checking the intrinsic fluctuations with respect to the possible pionic angular momenta L for fixed K_3, eq.(16), we deduce, that $L = 0$ can only occur for

(i) $K_3 = 0$ in the third component $M = 0$ of the body-fixed torus fields $\mathbf{n}$, or

(ii) $\mid K_3 \mid = 2$ for the first two components of $\mathbf{n}$ with $\mid M \mid = 1$.

These two possibilities exhaust the set of πNN incoming channels to which all others are coupled according to eq.(18).

Consider case (i) now: since the incoming two-nucleon spin Σ is zero, the parity of the incoming rotational motion must be positive, because $\mid N_3 \mid \leq \Sigma = 0$. The negative parity of the incoming channel must be attributed to the vibrational excitation: so here the incoming channel is based on a $K_3^P = 0^-$ vibration. From Table 1 we conclude, however, that the multipole decomposition of the $K_3^P = 0^-$ vibration begins only above $L \geq 2$. In fact, we see that given the corresponding form of $\mathbf{s}(\mathbf{x})$ we have

$$(\mathbf{s}(\mathbf{x}) \cdot \nabla)_{K_3=0}^{P=-1} \mathbf{n}(\mathbf{x}) = z\ \partial_\varphi \begin{pmatrix} \cos 2\varphi\ \sin\alpha(r,\theta) \\ \sin 2\varphi\ \sin\alpha(r,\theta) \\ \cos\alpha(r,\theta) \end{pmatrix} \quad , \tag{23}$$

the components of which can be expressed in terms of the $Y^2_{\pm 2}$ spherical harmonics since the third component (M=0) vanishes. So case (i) does not couple to the incoming channel required by the experimental setup.

This leaves case (ii) as unique possibility. Since the intrinsic vibration in the incoming $L = 0$ channel must have $\mid M \mid = 1$ we also must have $\mid N_3 \mid = 1$ for an incoming target state of $\Sigma = 0$. Hence total negative parity stems from the incoming target state and the intrinsic vibration must carry $\mid K_3 \mid^P = 2^+$. This intrinsic vibration therefore just corresponds to a mode, which tends to separate the torus into two $B = 1$ solitons and which turns out to be rather low in energy. We have estimated its energy using the scaling ansatz discussed earlier and the basis for the displacement fields given in Table 1. We have multiplied each element of the basis by an arbitrary polynomial in r and z^2 searching for minimal excitation energy. In this way we obtained the estimate $\omega = 250$ MeV, which is 110 MeV above pion threshold. In this estimate we have used the standard Skyrme model parameters for scattering in the $B = 1$ sector: $f_\pi = 93$ MeV, $m_\pi = 138$ MeV, $e = 4$.

Even if one relaxes the initial constraint: incoming pions in an $L = 0$ channel, one can show by analogue arguments that $L = 1$ channels cannot couple to $J^P = 0^-$ reactions on two nucleons. Finally, $L \geq 2$ channels require $\Sigma \geq 2$ target states, inaccessible from two-nucleon states. Thus the low-lying separation mode of the torus is the door-way channel via which a $J^P = 0^-$ scattering process on two nucleons must proceed.

Till now we have left aside all considerations with respect to isospin, to which we turn now. Since the incoming channel specified in case (ii) must have $\mid N_3 \mid = 1$ the Clebsch-Gordon coefficient in the channel decomposition of the scattering states, eq.(18), forces total isospin I to be greater than zero leaving $I = 1, 2$ for incoming channels containing two nucleons. The case of $I = 1$ has been ruled out in the interpretation of the scattering experiment because it leads to a large decay width to the channel $\pi NN \rightarrow NN$, leaving only $I = 0, 2$ as possibilities. Thus, the Skyrme

model scattering states of pions on two nucleons would coincide with the experimental determination of scattering quantum numbers only if total isospin I is uniquely $I = 2$. Having clarified the role of the separation mode of the torus in $J^P = 0^-$ scattering of a pion on two nucleons, we still have to convince ourselves, that this mode will actually lead to a narrow resonance roughly 50 MeV above pion threshold. In principle, one would have to calculate the pion-torus scattering amplitudes the same way as it had been done for pion-soliton scattering[10, 11], the latter with remarkable success for those channels, where higher order contributions in $1/N_C$ are unimportant. Avoiding this formidable effort, at least for the moment, we resort to estimates. One estimate presented here was based on the eigenfrequencies of prescribed scaling modes, which for the door-way vibration under consideration came out at 110 MeV above pion threshold. Alternatively, one may consult the two-soliton calculations for the deuteron based on the Atiyah-Manton ansatz[14], which find the separation mode as softest mode - apart from the zero modes, of course - at 130 MeV. However, latter calculation could only be performed for massless pions, so the relation between the two estimates given is somewhat unclear. As for the width we have no handle for the moment, apart from actually doing the pion-torus scattering. But, if a resonance will emerge at roughly the estimated position it must have a small width, because it is a resonance close to threshold. We conclude by emphasizing that within the approximations and version of the $SU(2)$ Skyrme model used here, the isospin assignment $I = 0$ for a $J^P = 0^-$ dibaryon resonance seems to be excluded given the setup used in the experiments. A resonance with isospin $I = 2$, however, appears very naturally in form of a separation mode of the underlying toroidal $B = 2$ soliton configuration.

We wish to thank Niels Walet for explaining to us the structure of his lowest vibrational mode, i.e. the separation mode of the torus. This work was initiated while we were participating at the INT-95-1 session on “Chiral Dynamics in Hadrons and Nuclei” at the Institute for Nuclear Theory at the University of Washington, USA. We wish to thank the organizers of the session for the invitation to participate and the Department of Energy, USA for partial financial support during that period. One of us, B.S., wishes to thank the Istituto Nazionale di Fisica Nucleare for its hospitality allowing us to complete the investigation.

References

[1] R. Bilger, H. A. Clement and M. G. Schepkin, Phys. Rev. Lett. **71** (1993) 42.

[2] R. Bilger, H. A. Clement and M. G. Schepkin, Phys. Rev. Lett. **72** (1994) 2972.

[3] H. Garcilazo and L. Mathelitsch, Phys. Rev. Lett. **72** (1994) 2971.

[4] L. Ya Glozman, A. Buchmann and A. Faessler, J. Phys. **G20** (1994) L49; G. Wagner, L. Ya Glozman, A. Buchmann and A. Faessler, nucl-th/9502028.

[5] V. B. Kopeliovich and B. E. Stern, JETP Lett. **45** (1987) 203, Pisma v ZhETF **45** (1987) 165.

[6] J. J. M. Verbaarschot, Phys. Lett. **B195** (1987) 235.

[7] R. A. Leese, N. S. Manton and B. J. Schroers, Nucl. Phys. **B442** (1995) 228.

[8] N.R. Walet, Nucl. Phys. **A586** (1995) 649.

[9] B. Schwesinger, Baryon-baryon interactions in large N_C chiral perturbation theory, Siegen University preprint SI-95-TP3S1, hep-ph/9504421, submitted to Phys. Rev. **D**.

[10] H. Walliser and G. Eckart, Nucl. Phys. **A429** (1984) 514.

[11] A. Hayashi, G. Eckart, G. Holzwarth and H. Walliser, Phys. Lett. **B147** (1984) 5.

[12] H. Weigel, B. Schwesinger and G. Holzwarth, Phys. Lett. **B168** (1986) 321.

[13] A. Hayashi and G. Holzwarth, Phys. Lett. **B140** (1984) 175.

[14] M. F. Atiyah and N. S. Manton, Phys. Lett. **B222** (1989) 438.

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Table 1

$\mid K_3 \mid^P = 0^-$	$\begin{pmatrix} y\ z \\ -x\ z \\ 0 \end{pmatrix}$
$\mid K_3 \mid^P = 2^+$	$\begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} \quad ; \quad (x^2-y^2)\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \quad ; \quad xy\begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} \quad ; \quad (x^2-y^2)\begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$

[Table 1:] Basic building blocks for the displacement fields $\mathbf{s}(\mathbf{x})$ in the $\mid K_3 \mid^P = 0^-$ and 2^+ channels, as explained in the text. These building blocks are obtained using eq.(11) for the lowest possible values of K, disregarding all combinations which decouple from them.